E 4397 (Pages : 3) Reg. No.....

Name.....

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, OCTOBER 2012

Fifth Semester

Core Course—MATHEMATICAL ANALYSIS

(Common for Model I and Model II B.Sc. Mathematics and B.Sc. Computer Applications)

Time : Three Hours Maximum Weight : 25

Part A (Objective type questions)

Answer **all** *the questions.*

Each bunch of **four** *questions has weight* 1.

Bunch I

1. Find the supremum and infimum of the set $\left\{1-\frac{(-1)^n}{n} : n \in \mathrm{N}\right\}$.
2. Prove or disprove : The greatest member of a set always belong to the set.
3. Give an example of a set which is neither open nor closed.
4. Give an example of a perfect set.

Bunch II

5. Give an example of a nowhere dense set in $\mathbb{R}$.
6. Prove or disprove : Subset of an uncountable set is uncountable.
7. Give an example of a closed set which is not an interval.
8. Give an example of a non empty set whose derived set is empty.

Bunch III

9. If possible give an example of a sequence of real numbers (a_n) such that $\lim a_n \neq \overline{\lim}\, a_n$.
10. Give an example of a sequence which is not Cauchy.
11. If possible give examples of two sequences which are not converges but its sum is a convergent sequence.
12. If (a_n) and (b_n) are two sequences such that $a_n < b_n$. Let $a = \lim a_n$ and $b = \lim b_n$. What is the relation between a and b ?

Bunch IV

13. Prove or disprove : The product of two divergent sequences is a divergent sequence.
14. Give an example of a monotonic sequence.

Turn over

15. Write the relation between Re (iz) and $Im\, z$.

16. Give an example of a connected subset of $\mathbb{C}$.

$(4 \times 1 = 4)$

Part B (Short Answer Questions)

Answer any **five** *questions.*

Each question has weight 1.

17. If $S < T \subseteq \mathbb{R}$, where $S \neq \phi$, then show that if T is bounded below then inf $T \leq$ inf S.

18. State Dedikind's property of real numbers.

19. Show that the interior of $\mathbb{R}$ is R.

20. Show that for every set S, the closure $\bar{S}$ is closed.

21. Show that $\lim_{n \to \infty} \frac{1+2+3+.....+n}{n^2} = \frac{1}{2}$.

22. Show that for any real number $x, \lim \frac{x^n}{n!} = 0$.

23. Show that $\left(1 \cdot \frac{2}{1} \cdot \frac{3}{2} \cdot \frac{4}{3} \cdot \cdots\cdots \frac{n}{n-1}\right)^{\frac{1}{n}} \to 1$ as $n \to \infty$.

24. Prove that z is either real or purely imaginary iff $\bar{z}^2 = z^2$.

$(5 \times 1 = 5)$

Part C (Short Essay Questions)

Answer any **four** *questions.*

Each question has weight 2.

25. Show that every open onterval $]\, a, b\, [$ contains a rational number.

26. Show that a set is closed iff its complement is open.

27. Show that a countable union of countable sets is countable.

28. State and prove Bolzano-Weirstrass Theorem for sequences.

29. State and prove Cesaro's theorem.

30. Show that if c is any n^{th} root of unity other than unity itself then $1 + c + c^2 + ...+ c^{n-1} = 0$.

$(4 \times 2 = 8)$

Part D (Essay Questions)

Answer any **two** *questions.*
Each question has weight 4.

31. Show that the Cartesian product of two countable sets is countable.

32. Show that :

(a) $\lim_{n \to \infty} \sqrt[n]{a} = 1$, if $a > 0$ and

(b) $\lim_{n \to \infty} \sqrt[n]{n} = 1$.

33. State and prove Cauchy's general principle of convergence.

$(2 \times 4 = 8)$